Extended affine root system (Simply-laced elliptic Lie algebras) (Lie Groups, Geometric Structures and Differential Equations : One Hundred Years after Sophus Lie)

数理解析研究所講究録
1150巻 2000年 p.70

Extended affine root system
(Simply-laced elliptic Lie algebras)

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Abstract

Let (R, G) be a pair consisting of an elliptic root system R with a marking G. Assume that the attached elliptic Dynkin diagram $\Gamma(R, G)$ is simply-laced (see Sect. 2). We associate three Lie algebras, explained in 1), 2) and 3) below, to the elliptic root system, and show that all three are isomorphic. The isomorphism class is called the elliptic algebra.

1) The first one is the subalgebra $\widetilde{\mathfrak{g}}(R)$ generated by the vacuum e^{α} for $\alpha \in R$ of the quotient Lie algebra $V_{Q(R)}/DV_{Q(R)}$ of the lattice vertex algebra (studied by Borcherds) attached to the elliptic root lattice $Q(R)$. This algebra is isomorphic to the 2-toroidal algebra and to the intersection matrix algebra proposed by Slodowy.

2) The second algebra $\widetilde{\mathfrak{e}}(\Gamma(R, G))$ is presented by Chevalley generators and generalized Serre relations attached to the elliptic Dynkin diagram $\Gamma(R, G)$. Since the Cartan matrix for the elliptic diagram has some positive off diagonal entries, the algebra is defined not only by Kac-Moody type relations but some others.

3) The third algebra $\widetilde{\mathfrak{h}}_{\mathrm{af}}^{\mathbb{Z}} * \mathfrak{g}_{\mathrm{af}}$ is defined as an amalgamation of a Heisenberg algebra and an affine Kac-Moody algebra, where the amalgamation relations between the two algebras are explicitly given. This algebra admits a sort of triangular decomposition in a generalized sense.

The first algebra $\widetilde{\mathfrak{g}}(R)$ does not depend on a choice of the marking G whereas the second $\widetilde{\mathfrak{e}}(\Gamma(R, G))$ and the third $\widetilde{\mathfrak{h}}_{\mathrm{af}}^{\mathbb{Z}} * \mathfrak{g}_{\mathrm{af}}$ do. This means the isomorphism depend on the choice of the marking i.e. on a choice of an element of $\mathrm{PSL}(2, \mathbb{Z})$.

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